Derivation of the distribution function for the overall scores based on two groups

The overall scores based on two sets of scores, one for each group is computed as follows

$$S = s^* \times \min(|s^1|, |s^2|)$$

where s^* is the sign of s^1 if $|s^1| < |s^2|$ and is the sign of s^2 otherwise.

We assume that s^1 and s^2 are independent and each has a standard normal distribution. Then

For $x \geq 0$,

$$
\begin{aligned}
P(S > x) &= \frac{1}{2} P(|s^1| > x, |s^2| > x) \\
&= 2 P(s^1 > x) P(s^2 > x) \\
&= 2(1 - \Phi(x))^2,
\end{aligned}
$$

where $\Phi(x)$ is the cumulative distribution function of standard normal distribution $N(0,1)$.

The distribution of S is

$$
F_S(x) = \begin{cases} 1 - 2(1 - \Phi(x))^2, & \text{if } x \geq 0 \\ 2(\Phi(x))^2, & \text{if } x < 0. \end{cases}
$$